A3 - Lösungsskizzen

1) $f_t(x) = x \cdot \ln x - t \cdot x$

$\underline{f_t'(x)} = 1 \cdot \ln x + x \cdot \frac{1}{x} - t = \underline{\ln x + 1 - t}$, $\underline{f_t''(x) = \frac{1}{x}}$

a) Nullstellen bei x_0: notw. u. hinr. Bed.: $f_t(x_0) = 0$

$x_0 \ln x_0 - t x_0 = 0 \Rightarrow x_0 (\ln x_0 - t) = 0 \qquad x_{01} = 0 \notin D_x$

$\ln x_0 - t = 0 \Rightarrow \ln x_0 = t \Rightarrow \underline{x_0 = e^t}$

b) Extremwert bei x_e: notw.: $f_t'(x_e) = 0 \Rightarrow \ln x_e + 1 - t = 0$

$\Rightarrow \ln x_e = t - 1 \Rightarrow \underline{x_e = e^{t-1}}$, $\underline{y_e} = e^{t-1} \cdot (t-1) - t e^{t-1} = -e^{t-1}$

hinr.: $f_t''(x_e) \neq 0 \quad f_t''(e^{t-1}) = \frac{1}{e^{t-1}} > 0 \Rightarrow$ Minimum bei x_e

$\underline{T_t (e^{t-1} \mid -e^{t-1})}$

c) Wendepunkt bei x_w notw. $f_t''(x_w) = 0 \Rightarrow \frac{1}{x_w} = 0$ keine Lösung!

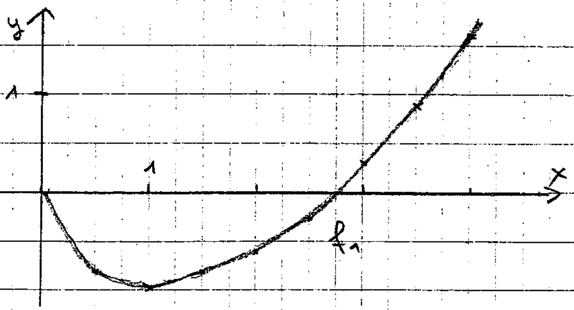
d) $\lim\limits_{x \to 0} (x \cdot \ln x - t \cdot x) = \lim\limits_{x \to 0} x \cdot \ln x - \lim\limits_{x \to 0} t \cdot x = 0 - 0 = 0$

Begründung: $\lim\limits_{x \to 0} x \cdot \ln x = \lim\limits_{x \to 0} \frac{\ln x}{\frac{1}{x}} \underset{\text{L'Hospital}}{=} \lim\limits_{x \to 0} \frac{\frac{1}{x}}{-\frac{1}{x^2}}$

$= \lim\limits_{x \to 0} (-x) = 0$

2) $f_1(x) = x \cdot \ln x - x$

x	[0]	0,5	1	1,5	2	2,5	3	$3\frac{1}{2}$	4
$f_1(x)$	[0]	-0,85	-1	-0,89	-0,61	-0,2	0,30	0,88	1,55



3) $T_t (e^{t-1} \mid -e^{t-1})$

$x = e^{t-1} > 0$, $y = -e^{t-1} = -x$

$\underline{y = -x \text{ und } x > 0}$ Ortskurve der Tiefpunkte

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4) a) $y_e = -e^{t-1} = -5 \Rightarrow t - 1 = \ln 5 \Rightarrow t = \ln 5 + 1 \approx 2{,}609$

b) $S_1 (e^{t-1} \mid -e^{t-1})$, $S_2 (e^t \mid 0)$ $\quad r = \sqrt{(x_{S_1} - x_{S_2})^2 + (y_{S_1} - y_{S_2})^2}$

$r = \sqrt{(e^{t-1} - e^t)^2 + (-e^{t-1})^2} = \sqrt{[e^{t-1}(1-e)]^2 + (e^{t-1})^2}$

$= e^{t-1} \sqrt{(1-e)^2 + 1} = e^{t-1} \sqrt{1 - 2e + e^2 + 1} = e^{t-1} \sqrt{2 - 2e + e^2}$

$r \approx e^{t-1} \sqrt{3{,}954} \approx 1{,}988\, e^{t-1}$

$r = 1 \Rightarrow e^{t-1} = \frac{1}{\sqrt{\quad}} \Rightarrow t - 1 = \ln \frac{1}{\sqrt{\quad}}$

$\Rightarrow t = \ln \frac{1}{\sqrt{2 - 2e + e^2}} + 1 \approx \underline{0{,}313}$

c) $P(3 \mid 4) \in f_t \Rightarrow 4 = 3 \cdot \ln 3 - 3t \Rightarrow t = \frac{3 \ln 3 - 4}{3} \approx -0{,}235$

5) HDI: $\left[x^2 \left(\frac{1}{2} \ln x - \frac{1}{2} t - \frac{1}{4} \right) \right]' = 2x \left(\frac{1}{2} \ln x - \frac{1}{2} t - \frac{1}{4} \right) + x^2 \frac{1}{2 \cdot x}$

$= x \cdot \ln x - tx - \frac{1}{2} x + \frac{1}{2} x = f_t(x)$ ✓

$J = \int (x \ln x - tx)\, dx = \int x \ln x\, dx - \int t x\, dx$

$J_1 = \int x \ln x\, dx$ partielle Integration $\quad u(x) = \ln x \Rightarrow u'(x) = \frac{1}{x}$, $v'(x) = x \Rightarrow v(x) = \frac{x^2}{2}$

$J_1 = \frac{x^2}{2} \cdot \ln x - \int \frac{x}{2}\, dx = \frac{x^2}{2} \ln x - \frac{x^2}{4} + c'$

$J = \frac{x^2}{2} \ln x - \frac{x^2}{4} - t \frac{x^2}{2} + c = x^2 \left(\frac{1}{2} \ln x - \frac{1}{2} t - \frac{1}{4} \right) + c$

6) Uneigentliches Integral $\int_0^{x_0} f_t(x)\, dx = \lim\limits_{a \to 0} \int_a^{x_0} f_t(x)\, dx$

$= \lim\limits_{a \to 0} \left[x^2 \left(\frac{1}{2} \ln x - \frac{1}{2} t - \frac{1}{4} \right) \right]_a^{e^t} = \lim\limits_{a \to 0} \left[(e^t)^2 \left(\frac{1}{2} t - \frac{1}{2} t - \frac{1}{4} \right) - \right.$

$\left. - \left(a^2 \left(\frac{1}{2} \ln a - \frac{1}{2} t - \frac{1}{4} \right) \right) \right] = -\frac{1}{4} (e^t)^2 = -\frac{1}{4} e^{2t}$

denn $\lim\limits_{a \to 0} \left[a^2 \left(\frac{1}{2} \ln a - \frac{1}{2} t - \frac{1}{4} \right) \right]$

$= \lim\limits_{a \to 0} \frac{1}{2} a^2 \ln a - \underbrace{\lim\limits_{a \to 0} a^2 \left(\frac{1}{2} t + \frac{1}{4} \right)}_{0} = 0$

$\lim\limits_{a \to 0} a^2 \cdot \ln a = \lim\limits_{a \to 0} \frac{\ln a}{\frac{1}{a^2}} \underset{\text{L'Hosp.}}{=} \lim\limits_{a \to 0} \frac{\frac{1}{a}}{-2 \frac{1}{a^3}} = \lim\limits_{a \to 0} \left(-\frac{1}{2} a^2 \right) = 0$

$\Rightarrow \underline{F} = |J| = \underline{\frac{1}{4} e^{2t}}$