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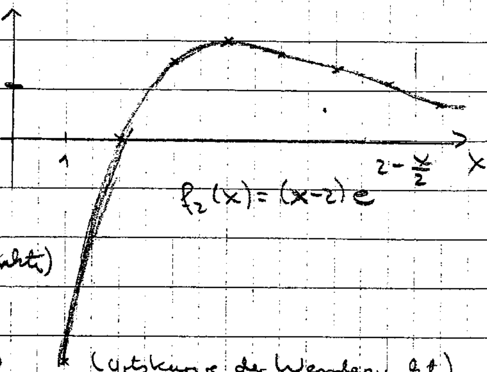
2)

x	1	2	3	4	5	6	7	8
$f_2(x)$	-4,48	0	1,65	2	1,82	1,47	1,12	0,81



3) $H_t(2t \mid t)$

$y = t$, $x = 2t$ ($x>0$, da $t>0$) $\quad t = \frac{x}{2}$

$\Rightarrow$ $y = \frac{x}{2}$ und $x>0$ (Ortskurve der Hochpunkte)

$W(3t \mid \frac{2t}{e})$ $\quad x = 3t$ ($x>0$) $\quad t = \frac{x}{3}$

$y = \frac{2}{e} \cdot t = \frac{2}{e} \cdot \frac{x}{3}$ $\quad y = \frac{2}{3e} \cdot x$ u. $x>0$ (Ortskurve der Wendepunkte)

4) a) w_t: $y = -\frac{1}{e} \cdot x + \frac{5t}{e}$ $\quad y = 0 \Rightarrow \frac{1}{e} x_s = \frac{5t}{e} \Rightarrow x_s = 5t$, $y_s = \frac{5t}{e}$

$F_\Delta(t) = \frac{1}{2} x_s \cdot y_s = \frac{25}{2e} \cdot t^2$

$F_\Delta(t) = 50 \Rightarrow \frac{25}{2e} \cdot t^2 = 50 \Rightarrow t^2 = 4e \Rightarrow \underline{t_1 = 2\sqrt{e}}$ $\approx 3,297$ $\quad (t_2 = -2\sqrt{e} \notin D_t)$

b) $x_s = x_0 = t \Rightarrow S_1(t \mid 0)$, $y_s = -te^2 \Rightarrow S_2 = (0 \mid -te^2)$

$|\overline{S_1 S_2}| = \sqrt{(x_{s_1} - x_{s_2})^2 + (y_{s_1} - y_{s_2})^2} = \sqrt{t^2 + t^2 e^4} = \underline{t \cdot \sqrt{1+e^4}}$

$t \cdot \sqrt{1+e^4} = 10 \Rightarrow \underline{t = \frac{10}{\sqrt{1+e^4}}} \approx 1,34$

5) HDI: $\left(-t \cdot x \cdot e^{2-\frac{x}{t}}\right)' = -te^{2-\frac{x}{t}} + (-t)xe^{2-\frac{x}{t}} \cdot \left(-\frac{1}{t}\right) = e^{2-\frac{x}{t}}(-t+x) \overset{= f_t(x)}{} $ ✓

oder (LK) Integrationsverfahren

$J_1 = \int e^{2-\frac{x}{t}} dx$ $\quad$ Substitution $u(x) = 2 - \frac{x}{t}$ $\quad \frac{du}{dx} = -\frac{1}{t} \Rightarrow dx = -t\,du$

$J_1 = \int e^u \cdot (-t) \cdot du = -t \int e^u du = -te^u + c_1^* = -te^{2-\frac{x}{t}} + c_1$

$J = \int (x-t)\, e^{2-\frac{x}{t}} dx$ $\quad$ partielle Integration

$u(x) = (x-t) \Rightarrow u'(x) = 1$

$v'(x) = e^{2-\frac{x}{t}} \Rightarrow v(x) = -te^{2-\frac{x}{t}}$ (vgl J_1!)

$J = -t \cdot (x-t) e^{2-\frac{x}{t}} + \int t e^{2-\frac{x}{t}} dx$ (und J_1!)

$= -t(x-t)e^{2-\frac{x}{t}} + t \cdot \left(-te^{2-\frac{x}{t}}\right)$

$= -te^{2-\frac{x}{t}}\left[(x-t) + t\right] = \underline{-t \cdot x \cdot e^{2-\frac{x}{t}}}$

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6) $J = \int_0^{x_0} f_t(x)\, dx = \left[-txe^{2-\frac{x}{t}}\right]_0^t = -t \cdot t\, e^{2-\frac{t}{t}} - 0$
$= \underline{-t^2 \cdot e}$

$F = |J| = \underline{t^2 \cdot e}$ FE

7) $\int_t^\infty f_t(x)\, dx = \lim\limits_{b \to \infty} \int_t^b f_t(x)\, dx$

$= \lim\limits_{b \to \infty} \left[-txe^{2-\frac{x}{t}}\right]_t^b = \lim\limits_{b \to \infty} \left[-t \cdot b\, e^{2-\frac{b}{t}} + t^2 e\right] = \underline{t^2 e}$

denn $\lim\limits_{b \to \infty} b\, e^{2-\frac{b}{t}} = \lim\limits_{b \to \infty} e^2 \cdot b\, e^{-\frac{b}{t}} = 0$ (vgl 1e!)

$t^2 e = e \Rightarrow t^2 = 1 \Rightarrow \underline{t_1 = 1}$ $\quad (t_2 = -1 \notin D_t)$