A5 – Lösungsskizzen – 3

6) $J_1 = \int \ln x \, dx$ partielle Integration

$u(x) = \ln x \Rightarrow u'(x) = \frac{1}{x}$

$v'(x) = 1 \Rightarrow v(x) = x$

$J_1 = x \cdot \ln x - \int \frac{1}{x} \cdot x \, dx = x \cdot \ln x - x + C_1$

$J = \int (\ln x)^2 dx$ partielle Integration

$u(x) = \ln x \Rightarrow u'(x) = \frac{1}{x}$

$v'(x) = \ln x \Rightarrow v(x) = x \cdot \ln x - x$

$J = x \cdot (\ln x)^2 - x \ln x - \int \ln x \, dx + \int 1 \, dx$

$= x (\ln x)^2 - x \ln x - x \ln x + x + x + C$

$= \underline{x (\ln x)^2 - 2x \ln x + 2x + C}$

$\int f_t(x) \, dx = t \int [\ln(x+t)]^2 dx$

Substitution $u(x) = x + t \Rightarrow \frac{du}{dx} = 1 \Rightarrow dx = du$

$\int f_t(x) dx = t \int (\ln u)^2 du = t \left[ (x+t) \cdot \ln(x+t)^2 - 2(x+t) \cdot \ln(x+t) + 2(x+t) \right] + $

7) $\int_{x_0}^{e-t} f_t(x) dx = \left[ F_t(x) \right]_{1-t}^{e-t}$

$= t \left( e \cdot (\ln e)^2 - 2e \cdot \ln e + 2e \right) - t \left( 1 \cdot (\ln 1)^2 - 2 \cdot 1 \cdot \ln 1 + 2 \cdot 1 \right)$

$= t e - t \cdot 2 = \underline{t(e-2)}$

$\underline{F(t) = t \cdot (e-2)}$ F.E.

Lösungsskizzen A6

$f_t(x) = 3x e^{-tx^2}$ ; $f_t'(x) = 3e^{-tx^2} + 3x e^{-tx^2} \cdot (-2tx)$

$f_t'(x) = 3e^{-tx^2} - 6tx^2 e^{-tx^2} = 3(1 - 2tx^2) e^{-tx^2}$

$f_t''(x) = -6tx e^{-tx^2} - 12tx e^{-tx^2} + 12 t^2 x^3 e^{-tx^2}$

$= 6tx (-3 + 2tx^2) e^{-tx^2}$

1) a) Symmetrie

$f_t(-x) = 3(-x) e^{-t(-x)^2} = -3x e^{-tx^2} = -f_t(x)$

<u>symmetrisch zum Ursprung</u>

b) Nullstellen bei $x_0$: notw. und hinr. $f_t(x_0) = 0$

$e^{-tx_0^2} \neq 0 \Rightarrow \underline{x_0 = 0}$

c) Extremwerte bei $x_e$: notw. $f_t'(x_e) = 0$.

$\Rightarrow 1 - 2tx_e^2 = 0 \Rightarrow x_e^2 = \frac{1}{2t}$ $\quad \underline{x_{e_{1,2}} = \pm \sqrt{\frac{1}{2t}}}$

$\underline{y_{e_{1,2}}} = \pm 3 \sqrt{\frac{1}{2t}} e^{-t \cdot \frac{1}{2t}} = \underline{\pm 3 \frac{1}{\sqrt{2te}}}$ $\quad \left( e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}} \right)$

hinr. $f_t''(x_e) \neq 0$

$f_t''\left(+\sqrt{\frac{1}{2t}}\right) = 6t \sqrt{\frac{1}{2t}} \overbrace{\left(-3 + 2t \cdot \frac{1}{2t}\right)}^{-2} e^{-t \cdot \frac{1}{2t}} = \frac{-12t}{\sqrt{2te}} < 0 \Rightarrow$ Max bei $x_{e_1}$

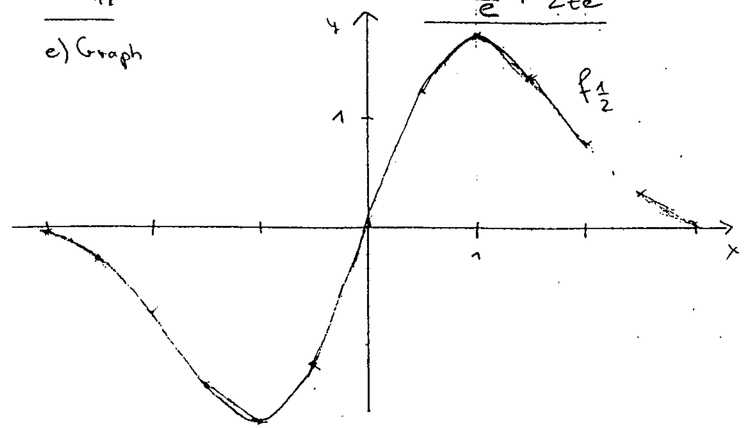
$H_t \left( \sqrt{\frac{1}{2t}} \middle| \frac{3}{\sqrt{2te}} \right)$ $\quad f_t''\left(-\sqrt{\frac{1}{2t}}\right) = \frac{12t}{\sqrt{2te}} > 0 \Rightarrow$ Min. bei $x_{e_2}$

d) Wendepunkte bei $x_w$: notw. $f_t''(x_w) = 0$

$\Rightarrow 2tx_w^2 = 3 \Rightarrow \underline{x_{w_{1,2}} = \pm \sqrt{\frac{3}{2t}}}$

$\underline{y_{w_{1,2}}} = \pm 3 \sqrt{\frac{3}{2t}} e^{-t \frac{3}{2t}} = \underline{\pm \frac{3}{e} \sqrt{\frac{3}{2te}}}$

e) Graph




$f_{\frac{1}{2}}(x) = 3x e^{-\frac{1}{2}x^2}$

| x | $f_{\frac{1}{2}}(x)$ |
|---|---|
| 0 | 0 |
| $\pm \frac{1}{2}$ | $\pm 1{,}3237$ |
| $\pm 1$ | $\pm 1{,}8196$ |
| $\pm 1\frac{1}{2}$ | $\pm 1{,}4609$ |
| $\pm 2$ | $\pm 0{,}8120$ |
| $\pm 2\frac{1}{2}$ | $\pm 0{,}3295$ |
| $\pm 3$ | $\pm 0{,}0999$ |