A 7 Lösungsskizzen -1-

1) $f_t(x) = (t - e^x) \cdot e^x = te^x - e^{2x}$

$f_t'(x) = t \cdot e^x - e^{2x} \cdot 2 = (t - 2e^x) \cdot e^x$

$f_t''(x) = t \cdot e^x - 2e^{2x} \cdot 2 = (t - 4e^x) \cdot e^x$

allgemeiner Beweis: zu zeigen $\left(f^{(n)}(x)\right)' = f^{(n+1)}(x)$

$\left[(t - 2^n \cdot e^x) e^x\right]' = \left[te^x - 2^n \cdot e^{2x}\right]' = t \cdot e^x - 2^n \cdot e^{2x} \cdot 2$

$= (t - 2^{n+1} \cdot e^x) \cdot e^x = f^{(n+1)}(x)$

Kurvendiskussion:

$D = \mathbb{R}$

Nullstellen bei x_0: notw. und hinr. $f_t(x_0) = 0$

$\Rightarrow (t - e^{x_0}) e^{x_0} = 0 \Rightarrow t \cdot e^{x_0} = 0 \Rightarrow t = e^{x_0} \mid \ln$

F.U. (1) $t > 0 \Rightarrow \underline{x_0 = \ln t}$ (2) $t \leq 0$ keine Nullst.

Extremwert bei x_e: notw.: $f_t'(x_e) = 0 \Rightarrow (t - 2e^{x_e}) e^{x_e} = 0$

$\Rightarrow t - 2e^{x_e} = 0 \Rightarrow e^{x_e} = \frac{t}{2} \mid \ln$ F.U. (1) $t \leq 0$ keine E.W.

(2) $t > 0 \Rightarrow \underline{x_e = \ln \frac{t}{2}}$, $\underline{y_e} = (t - \frac{t}{2}) \cdot \frac{t}{2} = \underline{\frac{t^2}{4}}$

hinr.: $f_t''(x_e) \neq 0$: $f_t''(\ln \frac{t}{2}) = (t - 4 \cdot \frac{t}{2}) \cdot \frac{t}{2} = -\frac{t^2}{2} < 0$

$\Rightarrow$ Maximum bei x_e $\underline{H_t \left(\ln \frac{t}{2} \middle| \frac{t^2}{4}\right)}$

Wendepunkt bei x_w: notw.: $f_t''(x_w) = 0 \Rightarrow$

$(t - 4e^{x_w}) e^{x_w} = 0 \Rightarrow e^{x_w} = \frac{t}{4} \mid \ln$ F.U. (1) $t \leq 0$ keine W

(2) $t > 0$ $\underline{x_w = \ln \frac{t}{4}}$ $\underline{y_w = (t - \frac{t}{4}) \cdot \frac{t}{4} = \frac{3}{16} t^2}$ $W_t \left(\ln \frac{t}{4} \middle| \frac{3}{16} t^2\right)$

Ortskurven:

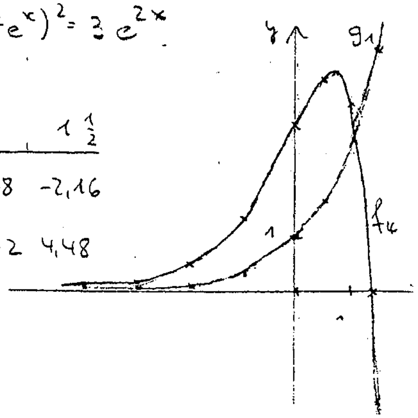
H_t: $x = \ln \frac{t}{2} \Rightarrow t = 2 \cdot e^x$; $y = \frac{t^2}{4} = \frac{1}{4} (2e^x)^2 = e^{2x}$

W_t: $x = \ln \frac{t}{4} \Rightarrow t = 4e^x$; $y = \frac{3}{16} (4e^x)^2 = 3e^{2x}$

Asymptote: $\lim_{x \to -\infty} f_t(x) = 0$ $A(x) = 0$

2)

x	-4	-3	-2	-1	0	$\frac{1}{2}$	1	$1\frac{1}{2}$
$f_4(x)$	0,07	0,20	0,52	1,34	3	3,88	3,48	-2,16
$g_1(x)$	0,02	0,05	0,14	0,37	1	1,65	2,72	4,48



A 7 Lösungsskizzen -2-

3) a) $A(0|2) \in f_t \Rightarrow f_t(0) = 2 \Rightarrow (t - e^0) \cdot e^0 = 2 \Rightarrow t - 1 = 2 \Rightarrow \underline{t = 3}$

b) $x_e = 5 \Rightarrow \ln \frac{t}{2} = 5 \Rightarrow \frac{t}{2} = e^5 \Rightarrow \underline{t = 2 \cdot e^5}$

c) $y_e = 9 \Rightarrow \frac{t^2}{4} = 9 \Rightarrow t^2 = 36 \Rightarrow \underline{t_1 = 6}$ ($t_2 = -6$ entfällt, kein E.W.)

d) $x_e - x_w = \ln \frac{t}{2} - \ln \frac{t}{4} = \ln t - \ln 2 - (\ln t - \ln 4) = \ln 4 - \ln 2$

$= 2 \cdot \ln 2 - \ln 2 = \ln 2$ $\underline{\text{für alle } t}$ ($t > 0$)

4) $F(a) = \int_a^{x_0} f_t(x)\,dx = \left[t \cdot e^x - \frac{1}{2} e^{2x}\right]_a^{\ln t} = \left(t \cdot t - \frac{1}{2} t^2\right) - \left(te^a - \frac{1}{2} e^{2a}\right)$

$= \frac{1}{2} t^2 - (t - \frac{1}{2} e^a) e^a$ $\lim_{a \to -\infty} (t - \frac{1}{2} e^a) e^a = 0$

$\Rightarrow \lim_{a \to -\infty} F(a) = \frac{1}{2} t^2$

5) Tangente bei x_0: $\frac{y - (e - e^{x_0}) \cdot e^{x_0}}{x - x_0} = (e - 2e^{x_0}) \cdot e^{x_0} \mid \cdot (-x_0)$

$A(0|e^2) \in g$: $e^2 - e \cdot e^{x_0} + e^{2x_0} = -e x_0 e^{x_0} + 2 x_0 e^{2x_0}$

$x_0 = 1 \Rightarrow e^2 - e^2 + e^2 = -e^2 + 2e^2$ ✓

6) $f_4 \cap g_k$: $(4 - e^{x_k}) e^{x_k} = k \cdot e^{x_k} \mid : e^{x_k} \Rightarrow 4 - e^{x_k} = k$

$e^{x_k} = 4 - k \mid \ln \overset{k<4}{\Rightarrow} x_k = \ln(4 - k)$ ggf.: $k > 0$

P_k im 1. Quadranten, falls $x_k > 0$, d.h. $\ln(4 - k) > 0$

also $4 - k > 1 \Rightarrow \underline{0 < k < 3}$ $y_k = k \cdot e^{x_k} > 0$

$F(k) = \int_0^{\ln(4-k)} g_k(x)\,dx = \left[k e^x\right]_0^{\ln(4-k)} = k \cdot (4 - k) - k = 3k - k^2$

$F'(k) = 3 - 2k$, $F''(k) = -2$

Extremwert bei k_e: notw.: $F'(k_e) = 0 \Rightarrow k_e = \frac{3}{2}$

hinr.: $F''(k_e) \neq 0$, $F''(\frac{3}{2}) = -2 \Rightarrow$ Maximum bei $k_e = \frac{3}{2}$

7) $P_1 (\ln 3 | 3)$ $F_1 = \int_0^{\ln 3} f_4(x)\,dx = \left[4e^x - \frac{1}{2} e^{2x}\right]_0^{\ln 3} = 4 \cdot 3 - \frac{1}{2} \cdot 9 - (4 -$

$= 4$

$F_2 = \int_0^{\ln 3} g_1(x)\,dx = \left[e^x\right]_0^{\ln 3} = 3 - 1 = 2$

$\Rightarrow F_2 = \frac{1}{2} F_1$, d.h. g_1 halbiert die Fläche

$(F_1 - F_2) : F_2 = 1 : 1$